spectral equilibrium unilateral radiation flux; I, $\vec{S}$, integrated radiation intensity and flux with respect to the frequency; τ, optical density; θ, φ, angles; and α, β, formal parameters.

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QUESTION OF THE NONSTATIONARY RADIANT INTERACTION OF SOLIDS

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The nonstationary thermal interaction by the radiation of an unlimited plate and a source with a constant temperature is considered. A solution is obtained governing the temperature distribution in the plate with any previously assigned accuracy.

The solution of linear-heat-conduction equations with nonlinear boundary conditions is often required in practice. Thus, e.g., at high source temperatures the heat from the source is transmitted to a heated body mainly by radiation. In such cases the convective component turns out to be negligible. Problems of this kind occur every time bodies are heated or cooled in such a way that the convective heat flux is small compared to the radiant heat fluxes.

Despite the fact that the question of heat propagation in solids with radiant heat exchange on the boundaries is encountered quite often as a phenomenon, the number of investigations in the area of nonstationary heat conduction with radiation boundary conditions is, however, quite small [1-8, et al.]. This is evidently explained by the difficulty of the mathematical analysis. Moreover, the present lack of exact solutions of the problems mentioned makes investigation of the regularities of heat propagation in solids subjected to thermal radiation much more difficult.

Meanwhile, the problem of radiant heat transfer becomes more and more valuable in connection with the achievements in studying space and a number of other domains for which large temperature differences are characteristic.

An attempt is made in this paper to find the nonstationary temperature distribution in a plate which initially has the temperature T_0 and is suddenly subjected to the effect of radiation. Mathematically, the problem can be formulated as follows

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$$\frac{\partial T}{\partial \tau} = a_0 \frac{\partial^2 T}{dx^2} \quad (0 < x < R,\ \tau > 0), \tag{1}$$

$$T|_{\tau=0} = T_0 \qquad (0 \leqslant x \leqslant R), \tag{2}$$

$$T|_{x=0} = T_0 \qquad (\tau \geqslant 0), \tag{3}$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=R} = \frac{\sigma}{\lambda_0}\,(T_m^4 - T^4|_{x=R}) \qquad (\tau > 0), \tag{4}$$

where condition (4) reflects the known Stefan-Boltzmann law.

Let us assume that both the temperature T_m of the heating medium and all the thermophysical characteristics (a_0, σ, λ_0) are constant. Setting $\Theta = (T - T_0)/T_0$, $z = z/R$, $Fo = a_0\tau/R^2$, , $Sk = \sigma R T_0^3/\lambda_0$, we write the system of equations (1)-(4) thus;

$$\frac{\partial \Theta}{\partial \mathrm{Fo}} = \frac{\partial^2 \Theta}{\partial z^2}, \tag{5}$$

$$\Theta|_{\mathrm{Fo}=0} = 0, \tag{6}$$

$$\Theta|_{z=0} = 0, \tag{7}$$

$$\left.\frac{\partial \Theta}{\partial z}\right|_{z=1} = \mathrm{Sk}\left[\frac{T_m^4}{T_0^4} - (1 + 4\Theta + 6\Theta^2 + 4\Theta^3 + \Theta^4)|_{z=1}\right]. \tag{8}$$

Taking into account that the functions Θ^m (m = 1, 2, 3, 4) satisfy the Dirichlet conditions [9], then they can be represented in a certain temperature band (0, Θ_p) in the form of the Fourier series:

$$\Theta = \sum_{k=1}^{\infty} \alpha_k \sin\frac{k\pi\Theta}{\Theta_p}, \tag{9}$$

$$\Theta^2 = \sum_{k=1}^{\infty} \beta_k \sin\frac{k\pi\Theta}{\Theta_p}, \tag{10}$$

$$\Theta^3 = \sum_{k=1}^{\infty} \gamma_k \sin\frac{k\pi\Theta}{\Theta_p}, \tag{11}$$

$$\Theta^4 = \sum_{k=1}^{\infty} \delta_n \sin\frac{k\pi\Theta}{\Theta_p}, \tag{12}$$

where

$$\alpha_k = -\frac{2\Theta_p}{k\pi}\cos k\pi; \tag{13}$$

$$\beta_k = -\frac{2\Theta_p^2}{k\pi}\left[\left(1 - \frac{2}{(k\pi)^2}\right)\cos k\pi + \frac{2}{(k\pi)^2}\right]; \tag{14}$$

$$\gamma_k = -\frac{2\Theta_p^3}{k\pi}\left(1 - \frac{6}{(k\pi)^2}\right)\cos k\pi; \tag{15}$$

$$\delta_k = -\frac{2\Theta_p^4}{k\pi}\left[\left(1 - \frac{12}{(k\pi)^2} + \frac{24}{(k\pi)^4}\right)\cos k\pi - \frac{24}{(k\pi)^4}\right]. \tag{16}$$

Taking into account (9)-(12) and the reduction rules [10], we represent the system (5)-(8) in the form

$$\frac{\partial T_k}{\partial \mathrm{Fo}} = \frac{\partial^2 T_k}{\partial z^2}, \tag{17}$$

$$T_k|_{\mathrm{Fo}=0} = 0, \tag{18}$$

$$T_k|_{z=0} = 0, \tag{19}$$

$$\left.\frac{\partial T_k}{\partial z}\right|_{z=1} = M_k - N_k T_k|_{z=1}, \tag{20}$$

where

$$T_k = \alpha_k \sin\frac{k\pi\Theta}{\Theta_\text{п}}; \tag{21}$$

$$M_k = \mathrm{Sk}\frac{1}{e(k-1)!}\left(\frac{T_\text{c}^4}{T_0^4} - 1\right); \tag{22}$$

$$N_k = \mathrm{Sk}\frac{4\alpha_k + 6\beta_k + 4\gamma_k + \delta_k}{\alpha_k}, \tag{23}$$

and e is the base of the natural logarithms. Therefore, a linear system of equations (17)-(20) has been obtained which is solved sufficiently simply. Applying the Laplace transform, we find that

$$\bar{T}_k = \frac{M_k}{s}\,\frac{\mathrm{sh}\sqrt{s}\,z}{\sqrt{s}\,\mathrm{ch}\sqrt{s} + N_k\,\mathrm{sh}\sqrt{s}}, \tag{24}$$

from which by going back to the original

$$T_k = \frac{M_k}{1+N_k}z - 2M_k\sum_{n=1}^{\infty}\frac{\sin\mu_{nk}z}{(N_k + N_k^2 + \mu_{nk}^2)\sin\mu_{nk}}\exp[-\mu_{nk}^2\mathrm{Fo}], \tag{25}$$

where μ_{nk} are the roots of the characteristic equation

$$\mathrm{tg}\,\mu = -\frac{\mu}{N_k}. \tag{26}$$

Taking account of (9)-(21), after summing over k the final solution of the system of equations (5)-(8) will be

$$\Theta(z, \mathrm{Fo}) = \sum_{k=1}^{\infty}\frac{M_k}{1+N_k}z - 2\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{M_k\sin\mu_{nk}z}{(N_k + N_k^2 + \mu_{nk}^2)\sin\mu_{nk}}\exp[-\mu_{nk}^2\mathrm{Fo}]. \tag{27}$$

However, an unknown constant Θ_p which must be determined is left in the solution (27). In this connection, we find the solution of the system of equations

$$\frac{d^2\Theta_\text{c}}{dz^2} = 0, \tag{28}$$

$$\Theta_\text{c}|_{z=0} = 0, \tag{29}$$

$$\left.\frac{d\Theta_\text{c}}{dz}\right|_{z=1} = \mathrm{Sk}\left(\frac{T_\text{m}^4}{T_0^4} - 1\right) - \mathrm{Sk}\left.(4\Theta_\text{c} + 6\Theta_\text{c}^2 + 4\Theta_\text{c}^3 + \Theta_\text{c}^4)\right|_{z=1}. \tag{30}$$

From (28) and condition (29) we find that

$$\Theta_\text{c} = C_1 z. \tag{31}$$

Substituting solution (31) into condition (30), we obtain a fourth-order algebraic equation in C_1, i.e.,

$$C_1(1+4\mathrm{Sk})+6\mathrm{Sk}\,C_1^2+4\mathrm{Sk}\,C_1^3+\mathrm{Sk}\,C_1^4=\mathrm{Sk}\left(\frac{T_m^4}{T_0^4}-1\right) \tag{32}$$

or

$$C_1+\mathrm{Sk}\,(1+C_1)^4=\mathrm{Sk}\frac{T_m^4}{T_0^4}, \tag{33}$$

from which it is not difficult to find C_1 to any previously assigned accuracy.

Comparing the two stationary solutions (31) and (27), we obtain the transcendental equation

$$C_1 z=\sum_{k=1}^{\infty}\frac{M_k}{1+N_k}z, \tag{34}$$

from which Θ_p must be determined. Since C_1 and Θ_p can be determined to maximum accuracy, then the solution (27) can be considered exact.

NOTATION

$T(x, \tau)$, plate temperature at the time τ at a distance x from the origin; T_m = const, temperature of the heating medium; T_0, initial temperature; a_0, thermal diffusivity; λ_0, heat conduction; σ, Stefan–Boltzmann constant; Sk, Stark criterion; Fo, Fourier criterion; and Θ, relative temperature.

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